Electronically Measuring the Flight Time of Light

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Abstract. Attempting to measure the speed of light, we used a time amplitude converter to measure the time between the emission of a light signal by a light emitting diode and the detection of the light signal by a photomultiplier tube. Measurements of the voltage created by the time amplitude converter were taken first with a digital storage oscilloscope and subsequently with a multichannel analyzer. The slope of a line fit using the least-squares method of many measurements taken while varying the distance the light signal travels should approximate the speed of light. Our measurement of $(3.02 \pm 0.24) \times 10^8$ meters per second is in good agreement with the accepted value of 2.998×10^8 meters per second.

1 Introduction

Every form of electromagnetic radiation travels through a vacuum at the same speed, regardless of frequency or wavelength. In 1905, Albert Einstein proposed in his theory of special relativity that this speed was even constant regardless of the frame of the observer relative to the source, provided the reference frames are inertial. Ordinary objects with mass must also travel slower than the speed of light. The permittivity of free space and the magnetic constant (which appear frequently in the study of electromagnetism) are also related to the speed of light by the equation $c^2\epsilon_0\mu_0 = 1$, where ϵ_0 is the permittivity of free space and μ_0 is the magnetic constant.

In 1983, the value of the meter was redefined to make the speed of light exactly 299,792,458 meters per second (Tholen et al., 1983). Historically, however, the speed of light was one of the most studied - and measured - physical constants in science.

1.1 Background

Aristotle, an ancient Greek philosopher circa 350 BC, and Heron of Alexandria, an ancient Greek physicist and mathematician circa 60 AD, believed the speed of light to be infinite; that is, light reached its destination at the very instant it was emitted (Wikipedia, 2007). Early attempts at measuring the speed of light, while not very accurate or precise, proved that it was finite.

Several methods of measuring the speed of light produced astoundingly accurate results in the latter half of the 19th century and the early 20th century. These methods involved measurements of the speed of light propagating through air; this speed is very close to the speed of light through a vacuum, as the refractive index (the ratio of the speed of light through a vacuum to the speed of light through a given medium) of air is 1.0003.

Hippolyte Fizeau's attempt in 1849 used a rotating, notched wheel and a mirror thousands of meters away from a light source. Light shone on the rotating wheel and struck the mirror only when the wheel's cogs were not blocking it. The mirror reflected the light back at the rotating wheel, and an observer near the light source would detect the reflected light only when the wheel did not block it on its second pass, which occurred only at specific speeds of rotation. The speed of light through air could then be calculated, given this speed, the number of teeth on the wheel and the distances between the light source, mirror and observer. Fizeau concluded the speed of light must be around 313,000 kilometers per second (Fizeau, 1849).

Several subsequent improvements boosted the accuracy and precision of this method. Leon Foucalt replaced the rotating wheel by a rotating mirror, and in 1862 published the results of his measurement: 298,000 kilometers per second (Foucalt, 1862). Albert A. Michelson devoted much of his career to measuring the speed of light to great precision; in 1935, he used a rotating prism and a mirror more than 20 miles from a light source to measure the speed of light to be 299,794 $\pm$ 11 kilometers per second (Michelson et al., 1935).

After World War II, Louis Essen and A.C. Gordon-Smith used a microwave cavity to measure the speed of light. Their conclusion of 299,792 $\pm$ 3 kilometers per second was refined to 299,792.5 $\pm$ 1 kilometers per second by 1948 (Essen & Gordon-Smith, 1948).

In this report, we used a time amplitude converter (TAC) to measure the time of flight of a light pulse from a light emitting diode to a photomultiplier tube. The TAC outputs a voltage proportional to the time delay between triggering events. The TAC is triggered by a capacitor discharging which fires the light emitting diode and once again when the photomultiplier tube registers a drop in voltage caused by the incident light pulse. The speed of light was derived from the slope of a line fit by using the least squares method of measurements taken varying the distance light pulses must travel.

2 Methods and Materials

2.1 Required equipment

For this experiment, we used the following equipment:

- Time Amplitude Converter (TAC): Model 567 manufactured by EG&G Ortec
- Delay Module: nSec Delay model 2058 manufactured by Canberra

- Digital Storage Oscilloscope (DSO): Tektronics TDS 1002 (Dual channel digital storage oscilloscope)
- PC-based data acquisition card
- Multichannel analyzer software
- LED/capacitor module: Unknown manufacturer. Consists of a green LED and timer circuit. Cycles on and off at around 10KHz, depending on the voltage applied, which is around 200 volts DC
- LED Power Supply: Model 6207a manufactured by Harrison Industries (DC power supply, 0-200V, 0-0.2A)
- Photomultiplier Tube (PMT): Labeled N-134, unknown manufacturer, with magnetic shielding tube attached to the front of it
- PMT Power Supply: Model 315 manufactured by Bertan Associates, Inc. (DC power supply 0-5000V, 0-5mA)
- A long cardboard tube, about 15 centimeters in diameter and 5 meters long
- 3 meter sticks taped together
- Various BNC wires
- 2 Polarizing filters

2.2 Setup

As described in Dr. Gold's Junior Laboratory Manual Gold (2006), and clarified in Figure 1.

The LED module is connected to its power supply. It also has a BNC jack which has a voltage applied across it when the timing circuit turns on the LED. This BNC jack is connected to the first trigger input on the TAC. The LED module also has the meter sticks taped to it, and one of the polarizing filters is placed in the path of the LED. The module is inserted into one end of the cardboard tube so that the LED points down its length.

The PMT is connected to its power supply. Its anode is connected to the input on the delay module and to channel 1 of the oscilloscope. It has the other polarizing filter placed in front of its collecting end. The PMT is inserted into the opposite end of the long cardboard tube, with the collecting end pointed at the LED module.

The delay module's output is connected to the second trigger input on the TAC.

The TAC (which now has 2 inputs connected) has its output connected to channel 2 of the oscilloscope. In our subsequent measurements using a multichannel analyzer, channel 2 of

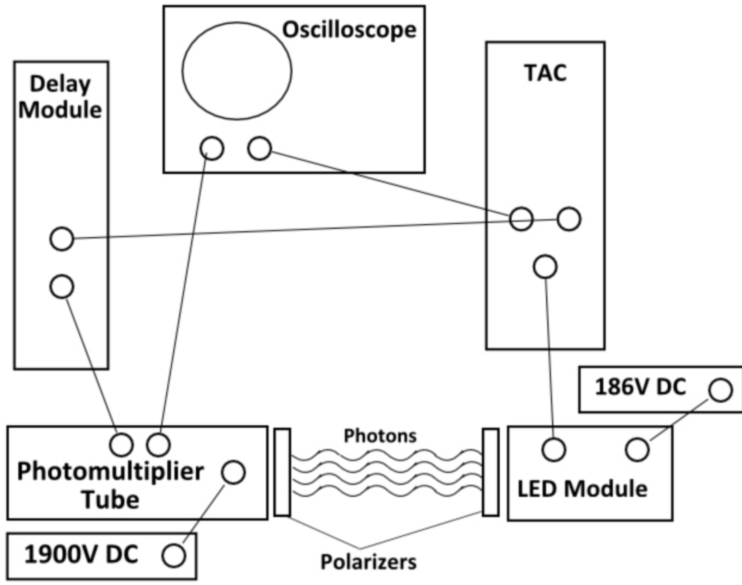

Figure 1 – *Setup diagram, with connections between instruments.*

the oscilloscope was split with a “T” connector and connected to the input of the PC data acquisition card.

The output will have a voltage across it proportional to the time in between trigger events. The constant of proportionality can be set with a knob on the face of the TAC. For this report, the TAC is set to output a voltage of 200 mV/ns.

The PMT power supply is set to around 1900 volts DC, and the LED power supply is set to around 186 volts DC.

2.3 Procedure

We began by turning on the power supplies, TAC and DSO. The LED module was firing at this point, and the PMT was registering a corresponding drop in potential for every pulse of incident light.

The LED module pulsing triggered the TAC to start. It was triggered again by a drop in potential across the photomultiplier tube caused by incident photons striking the photocathode material on the end of the PMT and the resulting cascade of electrons moving toward the anode. The TAC then created a potential across its two output leads which were proportional to the time between being triggered on and off. This voltage was measured with the oscilloscope and subsequently with the multichannel analyzer.

A very large source of systematic error must be accounted for: timewalk. Timewalk is an

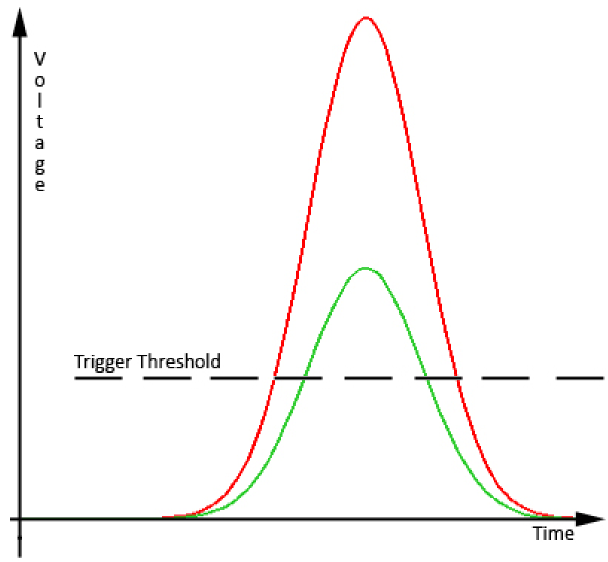

Figure 2 – *Illustration of timewalk. Note that both pulses begin and end at the same time, but the pulse with larger amplitude crosses the trigger threshold at an earlier time than the smaller amplitude pulse.*

interesting phenomenon which is explained very well in Dr. Gold's manual Gold (2006), but the essence is this: the TAC is triggered at a set voltage. This voltage will be reached sooner if the pulse being sent to the TAC is larger, and later if the pulse is smaller (see Figure 2). The size of the pulse is proportional to the brightness of the incident light on the PMT, which is proportional to the distance between the LED source and the PMT detector. To control this effect, a reference voltage was taken from the PMT which is roughly proportional to the brightness of the incident light. The polarizers in front of the source and emitter were turned as the distance changed in order to keep the brightness the same, indicated by the reference voltage.

By varying the distance between the LED module and the photomultiplier tube and taking voltage measurements, we determined the speed of light. Plotting the distance vs. time and taking the slope of the line connecting these points produced a rough estimate. By finding the slope of a line fit using the least-squares method, we obtained a better estimate.

We first used the oscilloscope to measure the TAC voltages (see Table 1). The voltages recorded are the result of reading channel 1 of the oscilloscope, which was the height of the square wave of potential produced by the TAC. This reading was steady, but the resolution of the oscilloscope was 0.02 volts, making our measurements imprecise.

We subsequently used the multichannel analyzer (MCA) to record the voltages produced by the TAC. The MCA software allowed us to record each instance of the LED firing and subsequent voltage produced by the TAC. Since the LED triggered at around 40 KHz, the MCA could

collect many hundreds of thousands of measurements in several seconds. The MCA software displayed a histogram of the voltages measured - a plot of a measurement vs. number of times that measurement was recorded. Unfortunately, the MCA software was not calibrated for our type of application and recorded measurements in arbitrary units. We determined these units by keeping the distance between the LED module and PMT constant and using the time delay box to delay the stop trigger by varied times.

3 Results and Discussion

3.1 Analysis

3.1.1 Oscilloscope Method

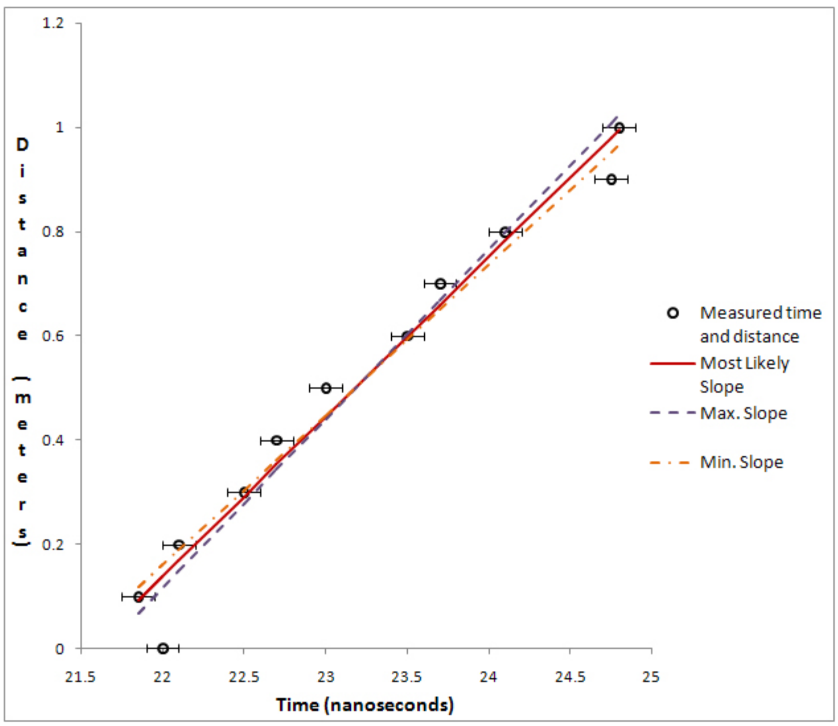

Figure 3 – *Oscilloscope Method: Plot of distance (in meters) from meter stick reading of 0.7m vs. measured times (in nanoseconds). The most likely slope, maximum and minimum slopes (which are one standard error higher and lower than the most likely slope, respectively) are also shown.*

The speed of light is the slope of a line fit by the least squares method. The line is of the form $y = mx + b$, where m is the slope and b is the y-intercept.

According to Taylor (1997), the slope of this line is

$$m = \frac{\sum x^2 \sum y - \sum x \sum xy}{\Delta}$$

and the y-intercept of the line is

$$b = \frac{N\sum xy - \sum x \sum y}{\Delta}$$

where $\Delta = N\sum x^2 - (\sum x)^2$ and N is the number of data.

The standard error of the slope is

$$\sigma_m = \sigma_y \sqrt{\frac{N}{\Delta}}$$

where $\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(y_i - b - mx_i)^2}$, and the standard error of the y-intercept is $\sigma_b = \sigma_y \sqrt{\frac{\sum x^2}{\Delta}}$.

In analyzing the data (see Table 1 in Addendum) from this experiment, the measured times are the x-values, and the distances are the y-values.

Thus, our most likely slope is 3.063×10^8 meters per second, and our most likely y-intercept is -6.60 meters. The standard error of the slope is 1.83×10^7 meters per second, and the standard error of the y-intercept is 4.24×10^{-1} meters. It's worth noting that the y-intercept is nonzero, which implies that even if the emitter and detector had no distance between them the TAC would still output a voltage. This is because the electrical signals travel through the BNC connections at some finite speed, and the cables are different lengths.

The most likely slope line is produced by pairing the most likely slope and most likely y-intercept. The maximum slope is the mean slope plus the standard error of the slope and the minimum slope is the mean slope minus the standard error of the slope. The maximum and minimum y-intercepts, similarly, are the mean y-intercept plus and minus the standard error of the y-intercept, respectively.

The maximum slope line comes and pairing the maximum slope and minimum y-intercept, while the minimum slope line is the pairing of the minimum slope and maximum y-intercept. Figure 3 is a plot of the data, most likely slope line, and maximum and minimum slope lines.

3.1.2 Multichannel Analyzer Method

Since the multichannel analyzer was not calibrated for our application, the measurements it recorded were in arbitrary units. We determined these units by adding a series of known time delays to the stop trigger and finding the slope of the line fit by the least squares method. These units were found to be (10.083 ± 0.089) bins per nanosecond. See Figure 4.

The slope of the line fit using the least squares method of our measurements varying the distance between the LED and PMT was calculated to be $(2.992 \pm 0.233) \times 10^{-2}$ meters per bin. See Figure 5.

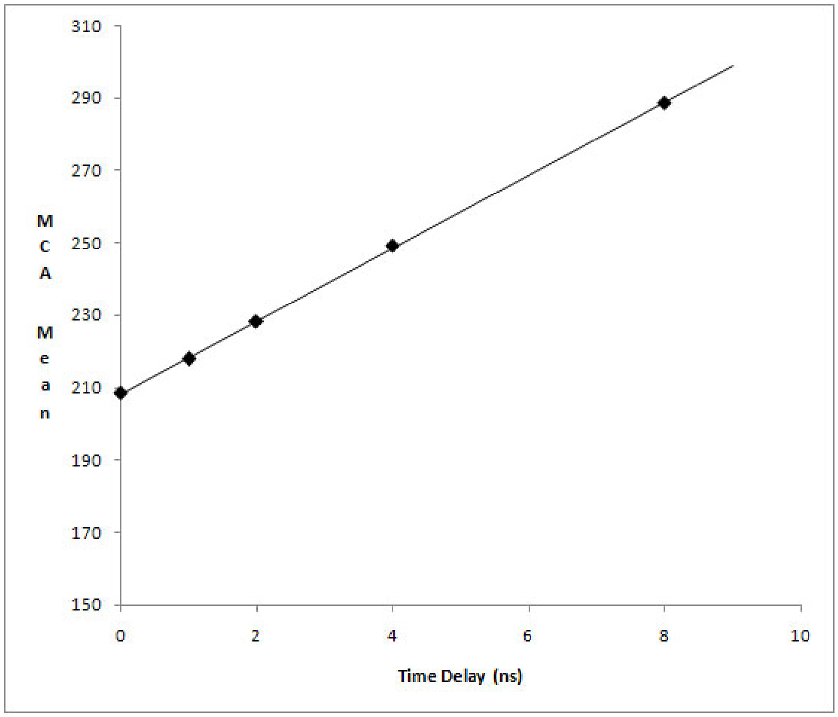

Figure 4 – *MCA method: Plot of MCA mean (in arbitrary units) vs. time delay (in nanoseconds). Since the time delay box should be very precise and accurate, a plot of data with very small residuals was expected (and seen).*

The product of the two slopes produces our measurement of the speed of light, which was $c = (3.02 \pm 0.24) \times 10^8$ meters per second. The uncertainty of our final product was calculated by adding the uncertainties of the two slopes in quadrature and multiplying by c; that is, $\frac{\delta c}{c} = \sqrt{(\frac{\delta k}{k})^2 + (\frac{\delta m}{m})^2}$, where k and δk represent the slope and uncertainty in slope of our measurements of time variation and m and δm represent the slope and uncertainty in slope of our measurements of distance variation.

3.1.3 Combining Methods

In order to combine the measurements of both methods and their uncertainties, we used a weighted mean of the measured speeds of light ("c"),

$$\bar{c} = \frac{\sum_{i=1}^{n} w_i c_i}{\sum_{i=1}^{n} w_i}$$

weighted by $w_i = \frac{1}{\sigma^2}$, where c_i is each speed of light measurement and σ_i is their uncertainty.

The uncertainty in this weighted mean was found by adding the uncertainties of the two methods in quadrature; that is, $\delta c = \sqrt{\delta c_1 + \delta c_2}$.

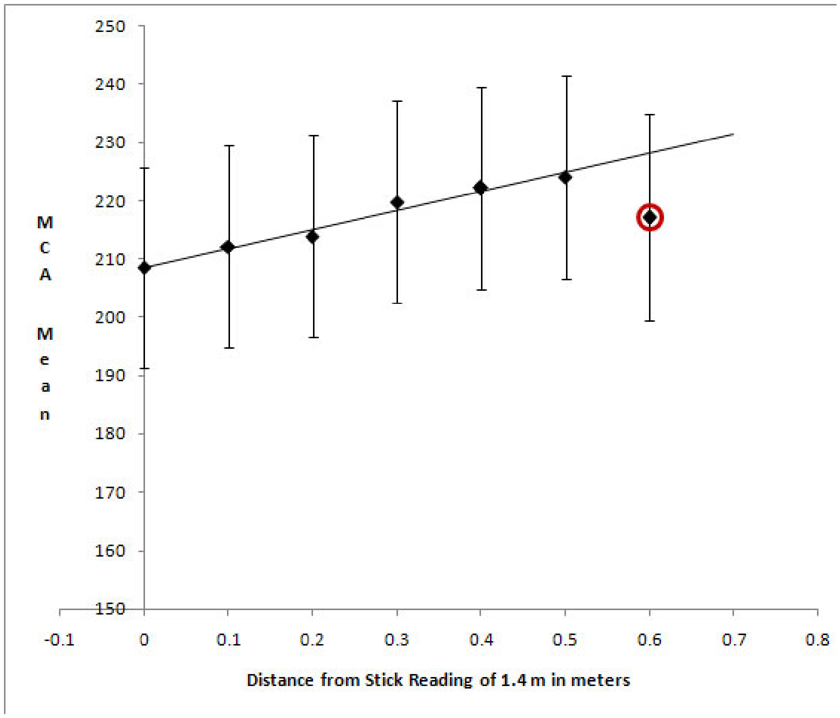

Figure 5 – *MCA method: Plot of MCA mean (in arbitrary units) vs. displacement of LED module (in m). The last datum (circled in red) was thrown out, justified by examining the residuals, their standard deviation and Chauvenet's criterion. The last point's residual was 2.59 standard deviations away from the regression value, which (assuming a Gaussian parent distribution) had a probability of being seen of less than 0.014 (Bevington & Robinson, 1992). Chauvenet's criterion states that the product of this probability and the number of measurements should be greater than 0.5 for valid data* ($N \times p > 0.5$) *(Taylor, 1997). This product was 0.11, which meets Chauvenet's criterion. (I'm not sure I applied this correctly, but I tried.)*

The weighted mean and uncertainty (our best estimate of the speed of light) are therefore $(3.05 \pm 0.3) \times 10^8$ meters per second.

4 Conclusions

While our result of $(3.05 \pm 0.3) \times 10^8$ meters per second is in good agreement with the accepted value of 2.998×10^8 meters per second, there was a relative uncertainty of 10%. The oscilloscope method produced a very accurate and more precise measurement than the MCA method did; however, if the MCA program were to be allowed to collect data for several minutes, instead of several seconds, the number of measurements of the voltage the TAC creates would grow very large and the standard error of the resulting distribution should be slightly smaller. Several more measurements using the MCA at different displacements of the LED could produce a measurement with less uncertainty, also.

A possible systematic source of error is the equipment. The reference voltage of the photomultiplier tube was inconsistent and varied from the recorded value by ± 4 millivolts to ± 8 millivolts. The cause for this inconsistency is uncertain; perhaps the LED module was not firing with a consistent voltage (and hence had a variable intensity).

5 Acknowledgements

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6 Addendum

6.1 Data

Oscilloscope Method - Measured Voltages					
Meter Stick Reading (in cm)	Distance from Reading of 140cm (in m)	Voltage	Error	Time (ns)	Error (ns)
40	1.0	4.96	± 0.02V	24.80	± 0.10
50	0.9	4.95	± 0.02V	24.75	± 0.10
60	0.8	4.82	± 0.02V	24.10	± 0.10
70	0.7	4.74	± 0.02V	23.70	± 0.10
80	0.6	4.70	± 0.02V	23.50	± 0.10
90	0.5	4.60	± 0.02V	23.00	± 0.10
100	0.4	4.54	± 0.02V	22.70	± 0.10
110	0.3	4.50	± 0.02V	22.50	± 0.10
120	0.2	4.42	± 0.02V	22.10	± 0.10
130	0.1	4.37	± 0.03V	21.85	± 0.15
140	0.0	4.40	± 0.02V	22.00	± 0.10

Table 1 – *These are measurements taken from the Time-Amplitude Converter using the dual channel oscilloscope. The voltages and their errors are the result of our best judgment by watching the Channel 1 "min" reading on the oscilloscope, set to average over 128 measurements. Typically, the Channel 1 minimum reading was unstable and varied between ± 0.02V or 0.03V, and seemed to spend most of the time around the recorded mean. This is, of course, not objective and could be a source of error. The third column ("Voltage") was the output of the function "min" for Channel 1 of the oscilloscope. The corresponding times in nanoseconds are the product of the voltage and 5 because the TAC was set to* $\frac{1}{5}$ *Volts per nanosecond. The errors of the times are related to the errors of the voltages by the expression* $\delta Time = 5 \times \delta Voltage$*, where* $\delta Voltage$ *and* $\delta Time$ *represent the uncertainty in voltage and time.*

MCA Method - Distance Variations, Measured Means			
Meter Stick Reading (in cm)	Distance from Reading of 140cm (in m)	MCA Mean (arbitrary units)	Standard Deviation (arbitrary units)
80	0.6	217.1	17.72
90	0.5	224.1	17.45
100	0.4	222.2	17.36
110	0.3	219.8	17.37
120	0.2	213.9	17.36
130	0.1	212.1	17.30
140	0.0	208.5	17.22

Table 2 – *No time delay, distance varied. These are measurements taken from the Time-Amplitude Converter using the multichannel analyzer. The means and standard deviations have arbitrary units because the MCA software was not designed for our application. We determined the unit by keeping the distance between the LED and PMT constant and delaying the stop trigger by known times.*

MCA Method - Time Variations, Measured Means		
Time Delay (ns)	MCA Mean (arbitrary units)	Standard Deviation (arbitrary units)
0	208.5	17.22
1	217.9	17.56
2	228.3	17.34
4	249.4	17.08
8	288.7	17.67

Table 3 – *Distance constant, time delay varied. These are measurements taken from the Time-Amplitude Converter using the multichannel analyzer. The means and standard deviations have arbitrary units because the MCA software was not designed for our application. We determined the unit by keeping the distance between the LED and PMT constant and delaying the stop trigger by known times.*